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Flow and Heat Transfer in a Viscoelastic Fluid Due To a Stretching Surface – An Analytical Solution Using Ham

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Abstract

In this study, asteady three dimensionalviscoelastic fluid flow and heat transfer due to a stretching sheet with an applied magnetic field is considered for analysis. The fluid far away from the surface is ambient and the motion in the flow field is caused by the stretching surface in two lateral directions.The heat transfer analysis has been carried out for two heating processes, viz. the constant wall temperature (CWT) and the constant heat flux (CHF). An analytical solution in the form of series expansionfor the velocity and temperature distribution is obtained employing homotopyanalysis method (HAM). The quantities of interest are velocity and temperature distribution as well as the skin-friction andthe wall heat flux. These are determined for various values of the parameters such as viscoelastic parameter, non-dimensional magnetic parameter, stretching ratio and the Prandtl number. The results obtained are presented in the form of graphs and tabulated data. It isobserved that both velocity and boundary layer thicknesses decrease withincreasing magnetic field. But the presence of magnetic parameter increases the heat transfer coefficients for both constant wall temperature and constant heat flux cases. The effects of the variation of all parameters discussed in detail. Thus, approximate analytical series solution obtained from Homotopy analysis is in very good agreement with the other methods and results.

Keywords:3D Viscoelastic fluid, Heat Transfer, Stretching Surface, Applied Magnetic field, HAM Solution

Introduction

In several engineering and mechanical processes such as extrusion, melt spinning etc., the extruded material issues through a die. The ambient fluid condition is stagnant, but the flow is induced near the material surface being extruded due to entrainment of the fluid caused by stretching of a moving surface. In regions away from the slit, the flow may be assumed to be of boundary layer type. However, this is not true near the slit. Similar situations arise during the manufacture of plastic and rubber sheets. Another example that belongs to the above class of problems is the cooling of large metallic plate in a bath, which may be an electrolyte. Glass blowing, continuous casting and spinning of fibers also involve the flow due to a stretching surface. In all these cases, the quality of the final product depends largely on the skin friction and heat transfer rates at the surface.

In recent years, the non-Newtonian fluids with magnetic field find increasing applications in industry and technology. The non-Newtonian fluids characterized by a power law model have some limitations as they do not exhibit any elastic property such as normal stress differences in shear flows. The equations of motion of the viscoelastic fluid are one order higher than the Navier-Stokes or the boundary layer equations governing the Newtonian fluids and the boundary conditions are not sufficient to determine the solution completely. Since the elastic parameter occurs as a coefficient of the highest derivative in the differential equations governing the flow, the mathematical problem reduces to a singular perturbation problem.

Sarpakaya [1] was mostly first researcher to investigate the magneto hydro dynamic flows of non-Newtonian fluids. Anderson[2]and Ariel[3] extended the above analysis to include the effect of the magnetic field. Dandapat and Gupta[4]andCortell[5] have considered the heat transfer aspect of this problem for the constant wall temperature case. Vajravelu and Rollins [6] have investigated the effect of variable surface temperature and surface heat flux on the heat transfer of a stretching surface. The three dimensional flow of Newtonian fluids over a stretching surface in two lateral directions without magnetic field was studied by Wang[7]. Unsteady three dimensional stagnation flow over a viscoelastic fluid was considered by Rajeswari et. al. [8].

Very recently in 2013, the three dimensional flow of Jeffrey fluid with variable thermal conductivity and thermal radiation was examined analytically by Hayat et.al [9]. Nandappanavaret. al.[10] studied the heat transfer of viscoelastic fluid flow due to nonlinear stretching sheet with internal heat source. They obtained approximate analytical local similar solutions of the highly non-linear momentum equation for velocity distribution by transforming the equation into Riccati-type and then solving this sequentially. The concept of Homotopy analysis method was first introduced in to the literature by Liao[11]. Following his idea of homotopy analysis, several hundred research appears are being published applying this technique. The homotopy solution for stagnation point flow of a non-Newtonian fluid an incompressible second grade fluid impinges on the wall was investigated by Hayat et.al [12].

The extension of [12] byShehzad et. al[13]presents the homotopy solution of boundary layer flow of Maxwell fluid with power law heat flux in the presence of heat source and hydromagnetics. Homotopy analysis method for MHD viscoelastic fluid flow and heat transfer in a channel with a stretching wall is examined by Raftari&Vajravelu[14].The flow of the viscoelastic fluid over a linearly stretching surface in an otherwise ambient fluid was investigated by Rajagopal at al.[15], who obtained the solution numerically for small values of the elastic parameter.

In all the above research papers, the combined effect of the three parameters such as applied magnetic field, stretching rate and the viscoelastic parameter on the velocity and temperature distribution and on skin friction and heat transfer rates have not been dealt with. Hence, the present analysis deals with the steady, three dimensional flow and heat transfer of a viscoelastic fluid over a stretching surface in two lateral directions with a magnetic field applied normal to the surface. The fluid far away from the surface is ambient and the motion in the flow field is caused by stretching surface in two directions.

Problem Formulation and Governing Equation

In reality, most of the fluids considered in industrial applications are non-Newtonian in nature, especially of viscoelastic type than viscous type. Hence, we consider the steady motion of a viscous incompressible electrically conducting viscoelastic fluid induced by the stretching of an infinite flat surface in two lateral directions x and y.The surface is assumed to be highly elastic and is being stretched by the action of uniform but increasing forces. Let 'a'and'b' be the rate of stretching in x and y directions, respectively. The fluid is assumed to have constant properties and it is at rest at infinity. No slip conditions are imposed on the stretching surface. The temperature of the ambient fluid is kept constant as T_∞. Both constant wall temperature case (CWT case) and constant heat flux case (CHF case) are included in the analysis. For the CWT case, the surface temperature is kept constant while for the CHF case the temperature gradient in the z- direction at the surface is kept uniform.

It is assumed that the normal stress is of the same order of magnitude as the shear stress. Thus both υ and λ_1 ($=\alpha_1/\rho$) are of $\delta^2$where δ is the boundary layer thickness. The magnetic field B is applied in z-direction. The magnetic Reynolds number Rm= $\mu_0\sigma VL<<1$, where $\mu_0$and σ are the magnetic permeability and electrical conductivity, respectively and V and L are the characteristic velocity and length. Hence the induced magnetic field is small in comparison to the applied magnetic field and is therefore neglected. The electrical current flowing in the fluid will give rise to an induced magnetic field if the fluid were an insulator. Here we have taken the fluid to be electrically conducting. The viscous dissipation and ohmic heating terms in the energy equation are neglected as they are assumed to be small. Under the foregoing assumptions, the boundary layer equations, governing the flow and the heat transfer of a viscoelastic fluid over a stretching surface in two lateral directions, are given by [2], [7] and [13].

$$u_x + v_y + w_z = 0 \quad (1)$$

$$uu_x + vu_y + wu_z = \upsilon u_{zz} - \frac{\sigma B^2 u}{\rho} + \lambda_1\left[\left(uu_{xzz} + vu_{yzz} + wu_{zzz}\right) - \left(u_{zz}u_x + v_{zz}u_y + w_{zz}u_z + 2u_z u_{xz} + 2v_z u_{yz} + 2w_z u_{zz}\right)\right] \quad (2)$$

$$uv_x + vv_y + wv_z = \upsilon v_{xzz} - \frac{\sigma B^2 v}{\rho} + \lambda_1\left[\left(uv_{xzz} + vv_{yzz} + wv_{zzz}\right) - \left(u_{zz}v_x + v_{zz}v_y + w_{zz}v_z + 2u_z v_{xz} + 2v_z v_{yz} + 2w_z v_{zz}\right)\right] \quad (3)$$

$$uT_x + vT_y + wT_{yy} = \left(\frac{k}{\rho Cp}\right)T_{zz} \quad (4)$$

The boundary conditions are given by:

$$u(x,y,0) = u_w\,;\quad v(x,y,0) = v_w\,;\quad w(x,y,0) = 0 \quad (5a)$$

$u(x,y,\infty) = v(x,y,\infty) = u_z(x,y,\infty) = v_z(x,y,\infty) = 0$(5b)

$T(x,y,\infty) = T_\infty$, $T(x,y,0) = T_w$for the CWT case; (5c)

$-k\left(\frac{\partial T}{\partial z}\right)_{z=0} = q_w$for the CHF case(5d)

Here x, y and z are the orthogonal coordinate system and u, v and w are the velocity components along x, y and z directions, respectively; ρand υ are the density and kinematic viscosityrespectively;λ_1 (=α_1/ρ)is the viscoelastic parameter; T is the temperature; Cp is the specific heat at constant pressure; k is the thermal conductivity; q_w is the heat transfer rate at the surface; a and b are the velocity gradients at the wall in x and y directions respectively; and the subscripts x,y,z denote derivatives with respect to x,y,z, respectively; and the subscripts w and ∞ denote conditions at the wall and in the ambient fluid, respectively. Also$u_w = ax$, $v_w = by$.

Eq.(1) to (4) can be reduced toa set of self-similar equations by apply the following transformations,

$$\eta = \left(\frac{a}{\upsilon}\right)^{1/2} z, u = axf'(\eta), \quad v = bys'(\eta),$$
$$w = -(a\upsilon)^{\frac{1}{2}}(f+cs), c = b/a$$

$M = \frac{\sigma B^2}{\rho a}andPr = \mu\left(\frac{Cp}{k}\right)$, $\lambda = \lambda_1\left(\frac{a}{\upsilon}\right) > 0$,

$T - T_\infty = (T_w - T_\infty)\, g(\eta)$for the CWT case,

$T - T_\infty = \left(\frac{q_w}{k}\right)\left(\frac{\upsilon}{a}\right)^{1/2} G(\eta)$for the CHF case,

It is verified that Eq.(1) is identically satisfied by the choice of u, v and w and Eq. (2)-(4) reduce to

$$f''' + (f+cs)f'' - (f')^2 - Mf' + \lambda[(f+cs)f'''' - 2(f' + cs')f''' + (f'' - cs'')f''] = 0$$
(7)

$$s''' + (f+cs)s'' - c(s')^2 - Ms' + \lambda[(f+cs)s'''' - 2(f' + cs')s''' + (cs'' - f'')s''] = 0$$
(8)

$g'' + Pr\,(f+cs)\,g' = 0$for the CWT

$G'' + Pr\,(f+cs)\,G' = 0$for the CHF(9b)

The boundary conditions Eqns. (5a-5d) are rewritten as,

$$f(0) = s(0) = 0, \quad f'(0) = 1, \ s'(0) = c,$$
$$f'(\infty) = s'(\infty) = f''(\infty) = s''(\infty) = 0:$$

$g(0) = 1$, $g(\infty) = 0$ for the CWT case

$G'(\infty) = -1$, $G(\infty) = 0$,for the CHF case
(10)

For Newtonian fluids (λ=0), the order the governing equation reduces and hence the conditions $f''(\infty) = s''(\infty) = 0$ are not required.

Here $\eta$is the transformed similarity variable; $f'$and ;$s'$are the dimensionless velocity components along x and y directions, respectively; g and G are the dimensionless temperatures for CWT and CHF cases, respectively; 'c' is the ratio of velocity gradients also known as stretching parameter; M is the magnetic parameter; $\mu$is the coefficient of viscosity;Pr is the Prandtl number; $\lambda$is the dimensionless viscoelastic parameter; and prime denotes derivative with respect to η. Eqn. (7) - (9) under conditions Eqn. (10) for λ=0 represent the three dimensional flow of Newtonian fluids and for λ>0 they represent the flow of non-Newtonian fluids (here it is viscoelastic fluids).

For Newtonian fluids the boundary conditions $f''(\infty) = s''(\infty) = 0$are not explicitly required. Also, Eq. (7) and (8) for λ= M = 0 are identical to those of Cortell [5]. For c=0, Eq. (7) - (9) (6) reduce to the two-dimensional flow over a stretching surface and for this case s = 0. Further, for c=1 these equations represent axisymmetric flow and in this case s=f. For c=0, Eq. (7) is identical to that of [2].

The skin friction coefficients in x and y directions can be expressed as

$C_{fx} = (1-\lambda)\,\mu\left(\frac{\partial u}{\partial z}\right)_{z=0} / \rho\, u_w u_w = (1-\lambda)\, Re_x^{-1/2} f''(0)$ (11a)

$C_{fy} = (1-\lambda)\,\mu\left(\frac{\partial v}{\partial z}\right)_{z=0} / \rho\, u_w v_w = (1-\lambda)\, Re_x^{-1/2} s''(0)$ (11b)

The heat transfer coefficient in terms of the Nusslet number can be written as,

$\mathrm{Nu} = \frac{x q_w}{k(T - T_\infty)}$ where $q_w = -\mathrm{k}\left(\frac{\partial T}{\partial z}\right)_{z=0} = -\mathrm{Re}_x^{\frac{1}{2}} g'(0)$ for CWT (12a)

$\mathrm{Nu} = -\mathrm{Re}_x^{\frac{1}{2}} / G'(0)$ for the CWT (12b)

Where C_{fx} and C_{fy} are the surface skin friction coefficients in x and y directions, Nu is the Nusslet number and $\mathrm{Re}_x (= ax^2/\upsilon)$ is the local Reynolds number.

Method of Solution

In order to obtain an analytical solution of the non-dimensional equations (7) to (9) satisfying the boundary conditions (10), we employ Homotopy Analysis Method (HAM). Thismethod is a recently developed approximate analytical method which is very popular amongst the present researchers. It is a special method in the sense that it is a pure analytic approach to solution procedure but solvable only using Computer Algebra System. This was introduced in the literature only in 1992 by S. J. Liao and is described in detail in[11]. Hence,the details are not presented here for the sake of brevity.However, the main components involved in applying HAM procedure are (i) selecting suitable initial profiles satisfying the boundary conditions of the problem and (ii) selecting an appropriate auxiliary linear operator so that its solutions are simpler to evaluate analytically.

In the present problem, depending upon the boundary conditions (10), we choose the initial guesses and auxiliary linear operators as follows.

$f_0(\eta) = 1 - e^{-\eta}, \quad s_0(\eta) = c(1 - e^{-\eta}), \quad g_0(\eta) = e^{-\eta}, \; G_0(\eta) = e^{-\eta}$ (13)

$L_f = \frac{d^3 f}{d\eta^3} - \frac{df}{d\eta}; \quad L_s = \frac{d^3 f}{d\eta^3} - \frac{df}{d\eta}; \quad L_g = \frac{d^2 f}{d\eta^2} - g; \quad L_G = \frac{d^2 f}{d\eta^2} - G$ (14)

so that,

$L_f[C_1 + C_2 e^{\eta} + C_2 e^{-\eta}] = 0$ (15)

$L_s[C_4 + C_5 e^{\eta} + C_6 e^{-\eta}] = 0$ (16)

$L_g[C_7 e^{\eta} + C_8 e^{-\eta}] = 0 \;; \quad L_G[C_9 e^{\eta} + C_{10} e^{-\eta}] = 0$ (17)

in which C_i are arbitrary constants. It is to be mentioned here that the choice of initial profiles and linear operators are not unique for a given problem, but the faster convergence of the solution depends on the choice of those.

The governing equations (7) to (9) prescribe the nonlinear operator for the homotopyanalysis through which a system of deformation equations are written.

Zeroth and Higher-order deformation Equations:

To obtain the HAM solution for the governing Eq. (7) - (9), in par with the standard notations followed in any HAM Analysis, let $\gamma \in [0, 1]$ be an embedding parameter and h_f, h_s and h_g are the non-zero auxiliary parameters. Then the non-linear operators and zeroth order deformation equation takes the form,

$$\mathcal{N}_f[\hat{f}(\eta,\gamma)] = \frac{\partial^3 \hat{f}(\eta,\gamma)}{\partial\eta^3} + \left(\hat{f}(\eta,\gamma) + c\hat{s}(\eta,\gamma)\right)\frac{\partial^2 \hat{f}(\eta,\gamma)}{\partial\eta^2} - \left(\frac{\partial \hat{f}(\eta,\gamma)}{\partial\eta}\right)^2 - M\left(\frac{\partial \hat{f}(\eta,\gamma)}{\partial\eta}\right) + \lambda\left(\left(\hat{f}(\eta,\gamma) + c\hat{s}(\eta,\gamma)\right)\frac{\partial^4 \hat{f}(\eta,\gamma)}{\partial\eta^4}\right) + \lambda\left(\frac{\partial^2 \hat{f}(\eta,\gamma)}{\partial\eta^2}\left(\frac{\partial^2 \hat{f}(\eta,\gamma)}{\partial\eta^2} - c\frac{\partial^2 \hat{s}(\eta,\gamma)}{\partial\eta^2}\right)\right) - 2\lambda\left(\frac{\partial^3 \hat{f}(\eta,\gamma)}{\partial\eta^3}\left(\frac{\partial \hat{f}(\eta,\gamma)}{\partial\eta} + c\frac{\partial \hat{s}(\eta,\gamma)}{\partial\eta}\right)\right) \quad (18)$$

$$\mathcal{N}_s[\hat{s}(\eta,\gamma), \hat{f}(\eta,\gamma)] = \frac{\partial^3 \hat{s}(\eta,\gamma)}{\partial\eta^3} + \left(\hat{f}(\eta,\gamma) + c\hat{s}(\eta,\gamma)\right)\frac{\partial^2 \hat{s}(\eta,\gamma)}{\partial\eta^2} - c\left(\frac{\partial \hat{s}(\eta,\gamma)}{\partial\eta}\right)^2 - M\left(\frac{\partial \hat{s}(\eta,\gamma)}{\partial\eta}\right) + \lambda\left(\left(\hat{f}(\eta,\gamma) + c\hat{s}(\eta,\gamma)\right)\frac{\partial^4 \hat{s}(\eta,\gamma)}{\partial\eta^4}\right) + \lambda\left(\frac{\partial^2 \hat{s}(\eta,\gamma)}{\partial\eta^2}\left(c\frac{\partial^2 \hat{s}(\eta,\gamma)}{\partial\eta^2} - \frac{\partial^2 \hat{f}(\eta,\gamma)}{\partial\eta^2}\right)\right) - 2\lambda\left(\frac{\partial^3 \hat{s}(\eta,\gamma)}{\partial\eta^3}\left(\frac{\partial \hat{f}(\eta,\gamma)}{\partial\eta} + c\frac{\partial \hat{s}(\eta,\gamma)}{\partial\eta}\right)\right) \quad (19)$$

$$\mathcal{N}_g[\hat{g}(\eta,\gamma),\hat{s}(\eta,\gamma),\hat{f}(\eta,\gamma)] = \frac{\partial^2\hat{g}(\eta,\gamma)}{\partial\eta^2} + Pr\left(\left(\hat{f}(\eta,\gamma)+c\hat{s}(\eta,\gamma)\right)\frac{\partial\hat{g}(\eta,\gamma)}{\partial\eta}\right)$$

$$(1-\gamma)L_f[\hat{f}(\eta,\gamma)-f_0(\eta)] = \gamma\hbar_f\mathcal{N}_f[\hat{f}(\eta,\gamma),\hat{s}(\eta,\gamma)] \quad (21)$$

$$(1-\gamma)L_s[\hat{s}(\eta,\gamma)-s_0(\eta)] = \gamma\hbar_s\mathcal{N}_s[\hat{f}(\eta,\gamma),\hat{s}(\eta,\gamma)] \quad (22)$$

$$(1-\gamma)L_g[\hat{g}(\eta,\gamma)-g_0(\eta)] = \gamma\hbar_g\mathcal{N}_g[\hat{f}(\eta,\gamma),\hat{s}(\eta,\gamma),\hat{g}(\eta,\gamma)] \quad (23a)$$

$$(1-\gamma)L_G[\hat{G}(\eta,\gamma)-G_0(\eta)] = \gamma\hbar_G\mathcal{N}_G[\hat{f}(\eta,\gamma),\hat{s}(\eta,\gamma),\hat{G}(\eta,\gamma)] \quad (23b)$$

Note that the nonlinear operator for CHF case is the same for the CWT case as they differ only in their boundary conditions.

Theboundary conditions take the form

$$\hat{f}(\eta,\gamma)|_{\eta=0}=0,\ \frac{\partial\hat{f}(\eta,\gamma)}{\partial\eta}|_{\eta=0}=1,\ \frac{\partial\hat{f}(\eta,\gamma)}{\partial\eta}|_{\eta=\infty}=0,\ \frac{\partial^2\hat{f}(\eta,\gamma)}{\partial\eta^2}|_{\eta=\infty}=0 \quad (24)$$

$$\hat{s}(\eta,\gamma)|_{\eta=0}=0,\ \frac{\partial\hat{s}(\eta,\gamma)}{\partial\eta}|_{\eta=0}=c,\ \frac{\partial\hat{s}(\eta,\gamma)}{\partial\eta}|_{\eta=\infty}=0,\ \frac{\partial^2\hat{s}(\eta,\gamma)}{\partial\eta^2}|_{\eta=\infty}=0 \quad (25)$$

$\hat{g}(\eta,\gamma)|_{\eta=0}=1$ *and* $\hat{g}(\eta,\gamma)|_{\eta=\infty}=0$ for CWT case (26a)

$\frac{\partial\hat{G}'(\eta,\gamma)}{\partial\eta}|_{\eta=0}=-1$ *and* $\hat{G}(\eta,\gamma)|_{\eta=\infty}=0$ for CHF case (26b)

For l th - order deformations equation, we first differentiate Eqns. (21)-(23) l times with respect to γ; dividing them by $l!$ and then set $\gamma=0$. Following this procedure we have

$$L_f[f_l(\eta)-\Omega_l f_{l-1}(\eta)] = \hbar_f\mathcal{R}_l^f(\eta) \quad (27)$$

$$L_s[s_l(\eta)-\Omega_l s_{l-1}(\eta)] = \hbar_s\mathcal{R}_l^s \quad (28)$$

$$L_g[g_l(\eta)-\Omega_l g_{l-1}(\eta)] = \hbar_g\mathcal{R}_l^g(\eta) \quad (29a)$$

$$L_G[G_l(\eta)-\Omega_l G_{l-1}(\eta)] = \hbar_G\mathcal{R}_l^G(\eta) \quad (29b)$$

With boundary conditions

(20)

$$f_l(0)=0,\ f_l{}'(0)=1,\quad f_l{}'(\infty)=0,\ f_l{}''(\infty)=0$$

$$s_l(0)=0,\ s_l{}'(0)=c,\ s_l{}'(\infty)=0,\quad s_l{}''(\infty)=0$$

$g_l(0)=1,\ g_l(\infty)=0$ for the CWT case

$G_l{}'(0)=-1,\ G_l(\infty)=1,$ for the CHF case (30)

Where $\mathcal{R}_l^f(\eta)$, $\mathcal{R}_l^s(\eta)$ *and* $\mathcal{R}_l^g(\eta)$ are remainder terms of the linear operators such as,

$$\mathcal{R}_l^f(\eta) = f_{l-1}{}'''(\eta) + \sum_{j=0}^{l-1}[f_{l-1-j}f_j'' - f_{l-1-j}'f_j'] + c\sum_{j=0}^{l-1}[s_{l-1-j}f_j''] + \lambda\sum_{j=0}^{l-1}[f_{l-1-j}f_j'''' + f_{l-1-j}''f_j''] + \lambda c\sum_{j=0}^{l-1}[s_{l-1-j}f_j'''' - s_{l-1-j}''f_j''] - 2\lambda\sum_{j=0}^{l-1}[f_{l-1-j}'f_j'''] - 2\lambda c\sum_{j=0}^{l-1}[s_{l-1-j}'f_j'''] - Mf_{l-1}'(\eta) \quad (31)$$

$$\mathcal{R}_l^s(\eta) = s_{l-1}{}'''(\eta) + \sum_{j=0}^{l-1}[f_{l-1-j}s_j'' - s_{l-1-j}'s_j'] + c\sum_{j=0}^{l-1}[s_{l-1-j}s_j''] + \lambda\sum_{j=0}^{l-1}[f_{l-1-j}s_j'''' + s_{l-1-j}''s_j''] + \lambda c\sum_{j=0}^{l-1}[s_{l-1-j}s_j'''' - s_{l-1-j}''s_j''] - 2\lambda\sum_{j=0}^{l-1}[f_{l-1-j}'s_j'''] - 2\lambda c\sum_{j=0}^{l-1}[s_{l-1-j}'s_j'''] - Ms_{l-1}'(\eta) \quad (32)$$

$$\mathcal{R}_l^g(\eta) = g_{l-1}''(\eta) + Pr\sum_{j=0}^{l-1}[f_{l-1-j}g_j'] + c\,Pr\sum_{j=0}^{l-1}[s_{l-1-j}g_j'] \quad (33a)$$

$$\mathcal{R}_l^G(\eta) = G_{l-1}''(\eta) + Pr\sum_{j=0}^{l-1}[f_{l-1-j}G_j'] + c\,Pr\sum_{j=0}^{l-1}[s_{l-1-j}G_j'] \quad (33b)$$

where Ω_l is defined as $\Omega_l = 0\ when\ l \le 1\ and\ \Omega_l = 1\ for\ l > 1$ (34)

Expanding $\hat{f}(\eta,\gamma), \hat{s}(\eta,\gamma)\ and\ \hat{g}(\eta,\gamma)$ in Taylor series with respect to γ,

$\hat{f}(\eta,\gamma) = f_0(\eta) + \sum_{l=1}^{\infty}[f_l(\eta)\gamma^l],\ f_l(\eta) = \frac{1}{l!}\frac{\partial^l\hat{f}(\eta;\gamma)}{\partial\gamma^l}|_{\gamma=0}$ (35) $\hat{s}(\eta,\gamma) = s_0(\eta) +$

$\sum_{l=1}^{\infty}[s_l(\eta)\gamma^l]$, $s_l(\eta) = \frac{1}{l!}\frac{\partial^l \hat{s}\,(\eta;\gamma)}{\partial\gamma^l}|_{\gamma=0}(36)\hat{g}(\eta,\gamma) = g_0(\eta) + \sum_{l=1}^{\infty}[g_l(\eta)\gamma^l]$, $g_l(\eta) = \frac{1}{l!}\frac{\partial^l \hat{g}\,(\eta;\gamma)}{\partial\gamma^l}|_{\gamma=0}$(37a)
$\hat{G}(\eta,\gamma) = G_0(\eta) + \sum_{l=1}^{\infty}[G_l(\eta)\gamma^l]$, $G_l(\eta) = \frac{1}{l!}\frac{\partial^l \hat{G}\,(\eta;\gamma)}{\partial\gamma^l}|_{\gamma=0}$(37b)

For$\gamma = 0$and $\gamma = 1$in Eq.(21)-(23) may be written as

$\hat{f}(\eta,0) = f_0(\eta), \qquad \hat{f}(\eta,1) = f(\eta)$(38)

$\hat{s}(\eta,0) = s_0(\eta), \qquad \hat{s}(\eta,1) = s(\eta)$(39)

$\hat{g}(\eta,0) = g_0(\eta), \qquad \hat{g}(\eta,1) = g(\eta)$(40a)

$\hat{G}(\eta,0) = G_0(\eta), \qquad \hat{G}(\eta,1) = G(\eta)$(40b)

Thus asγ increases from 0 to 1 and$\hat{f}(\eta,\gamma)$, $\hat{s}(\eta,\gamma)$,$\hat{g}(\eta,\gamma)$and $\hat{G}(\eta,\gamma)$varies from the initial guess, the functions$f_0(\eta)$, $s_0(\eta)$, $g_0(\eta)$and$G_0(\eta)$approaches to the solution $f(\eta)$, $s(\eta)$,$g(\eta)$and $G(\eta)$of the governing equations respectively. Here, theauxiliary parameters are suitably chosen so that the series solution converges for $\gamma = 1$.
$f(\eta) = f_0(\eta) + \sum_{l=1}^{\infty}[f_l(\eta)]$(41)

$s(\eta) = s_0(\eta) + \sum_{l=1}^{\infty}[s_l(\eta)]$(42)

$g(\eta) = g_0(\eta) + \sum_{l=1}^{\infty}[g_l(\eta)]$(43a)

$G(\eta) = G_0(\eta) + \sum_{l=1}^{\infty}[G_l(\eta)]$(43b)

Thereforewe get the general approximate analytical solutions (f_l, s_l, g_l *and* G_l) in terms of series solutions evaluated up tolth order (f_l^*, s_l^*, g_l^* *and* G_l^*) along with the solution of the linear operators chosen in the problem as

$f_l(\eta) = f_l^*(\eta) + \mathbf{C}_1 + \mathbf{C}_2 e^{\eta} + \mathbf{C}_3 e^{-\eta}$(44)

$s_l(\eta) = s_l^*(\eta) + \mathbf{C}_4 + \mathbf{C}_5 e^{\eta} + \mathbf{C}_6 e^{-\eta}$(45)

$g_l(\eta) = g_l^*(\eta) + \mathbf{C}_7 e^{\eta} + \mathbf{C}_8 e^{-\eta}$(46a)

$G_l(\eta) = G_l^*(\eta) + \mathbf{C}_7 e^{\eta} + \mathbf{C}_8 e^{-\eta}$(46b)

We solve the Eqns. (44) - (46) for various values of ‘l’ starting from $1, 2, 3, \ldots$ by means ofthe symbolic computation software Mathematica 8.0.

Computational Procedure

A Mathematica code is written to solve the above set of equations (35) to (40). Starting from known functions for$f_0(\eta)$, $s_0(\eta)$,$g_0(\eta)$ *and* $G_0(\eta)$, the next set of functions $f_1(\eta)$, $s_1(\eta)$, $g_1(\eta)$ *and* $G_1(\eta)$ are computed using the equations (35) to (40). Using these two sets, the next set of functions $f_2(\eta)$, $s_2(\eta)$, $g_2(\eta)$ *and* $G_2(\eta)$are algebraically computed and so on. At any stage, the functions obtained are a polynomial of degree l in ‘h’, the auxillaryhomotopy parameter, where the coefficients of the polynomial may contain some or all the parameters present in the problem. The computations are carried out up to l = 25, to ascertain that the values of $f_l(\eta)$, $s_l(\eta)$, $g_l(\eta)$ *and* $G_l(\eta)$ obtained at that stage are consistent and accurate up to 9 digits. It is to be mentioned here that the analytical expressions obtained for f_l^*, s_l^* *and* g_l^* are too lengthy even for the value of l = 5. Hence the actual expressions are reproduced here.

Convergence of the solutions

As it is done traditionally in anyhomotopy analysis, the auxiliary parameter $\hbar$ is optimized by drawing $\hbar$ -curves with respect to $\hbar_f$, $\hbar_s$and $\hbar_g$to find the convergent interval for f, s and g. From these curves it is found that the admissible ranges of $\hbar_f$, $\hbar_s$and $\hbar_g$ are $-1.3 < \hbar_f < 0$, $-1.2 < \hbar_s < 0$ and $-1.1 < \hbar_g < 0$. The $\hbar$curves are shown in Fig. 1. It is clear from Fig. 1 that the numerical values for f, s, g are insensitive to the value of $\hbar$ chosen fron the above mentioned range.

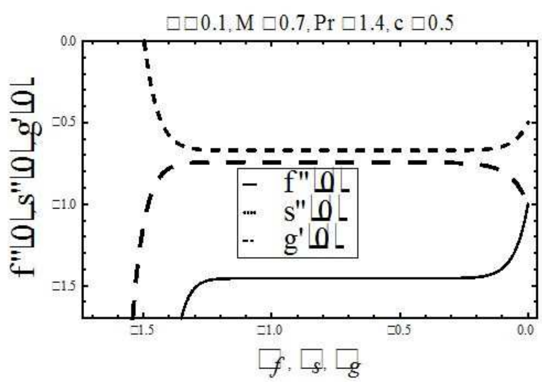

Figure. 1 $\hbar$**-curve for the functions f’’(η), s’’(η) and g’(η) at η = 0**

Results and Discussion

The analytical solutions of velocity and temperature profiles so obtained are a set of polynomials in higher powers of $\hbar$ (the homotopy parameter) whose coefficients contain M,the magnetic parameter; c, the stretching parameter andλ the viscoelastic parameter and

Pr, the Prandtl number. The solution can be obtained up to any order depending upon the required accuracy. It is often the case that the expressions obtained from HAM are too lengthy even for a polynomial of degree 5 in '$\hbar$'. In the present analysis, the computations have been performed up to 25th degree to ascertain the consistency in values, but the results presented in tables and figures are calculated from the 20th degree polynomial in '$\hbar$' of the series solution containing all the parameters.

In order to assess the accuracy of our results, we have compared our results for the Newtonian case ($\lambda = 0$) in the absence of magnetic parameter (M = 0) with[7] and [16] and presented in Table I for shear stress components in 'x' and 'y'. The same for the heat transfer case is compared with [17] to [18] in Table II. It is found that our results are in good agreement with the existing results.

c	$-f''(0)$ Wang[7]	$-f''(0)$ Ariel[16]	$-f''(0)$ Present	$-s''(0)$ Wang[7]	$-s''(0)$ Ariel[16]	$-s''(0)$ Present
0.00	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000
0.10	1.02090	1.01702	1.00199	0.06684	0.07309	0.06477
0.20	1.04180	1.03458	1.00787	0.14873	0.15823	0.14047
0.30	1.06270	1.05247	1.01745	0.24336	0.25434	0.22766
0.40	1.08360	1.07052	1.03056	0.34920	0.36059	0.32671
0.50	1.10450	1.08866	1.04704	0.46520	0.47629	0.43777
0.60	1.12541	1.10679	1.06670	0.59052	0.60083	0.56090
0.70	1.14631	1.12488	1.08935	0.72453	0.73373	0.69608
0.80	1.16721	1.14879	1.11481	0.86668	0.87455	0.84324
0.90	1.18811	1.16076	1.14287	1.01653	1.02292	1.00234
1.00	1.20901	1.17351	1.17334	1.17372	1.17851	1.17334

Table 1.Comparisonof skin friction results for steady case $\left(-f''(0),\ -s''(0)\right)$ when
Pr=1.3, $M = \lambda = 0$ and $\hbar_f = \hbar_s = \hbar_g = -0.6$.

Several computations have been made to check the influence of the parameters of study, such as λ, the viscoelastic parameter, 'c', the stretching ratio; 'M', the magnetic parameter on the velocity and temperature profiles as well as on skin friction and heat transfer coefficients.

Some of these results are presented here in the form of graphs. Figures 2(a), 2(b) and 2(c) show the influence of λ on the velocity and temperature profiles. It is observed that the boundary layer thickness decreases as the values of λ increases from 0.0 to 1.5, whereas the thermal boundary layer thickens for the increasing values of viscoelastic parameter. There is a temperature over shoot for the higher values of λas seen from figure 2c. Figures 3a, 3b, 3c present the effect of magnetic parameter, 'M' on velocity and temperature profiles and figures 4a, 4b and 4c shows the effect of the stretching ratio 'c',

c	$-g'(0)$ Kumari and G Nath [17]	$-g'(0)$ Ibrahim[18]	$-g'(0)$ Present HAM
0.00	0.4957	0.49587	0.494665
0.25	0.5358	0.53621	0.506619
0.50	0.5798	0.57967	0.540192
0.75	0.6233	0.62308	0.591103
1.00	0.6656	0.66538	0.654128

Table 2.Comparison of heat transfer results for steady case($-g'(0)$) when Pr = 0.7

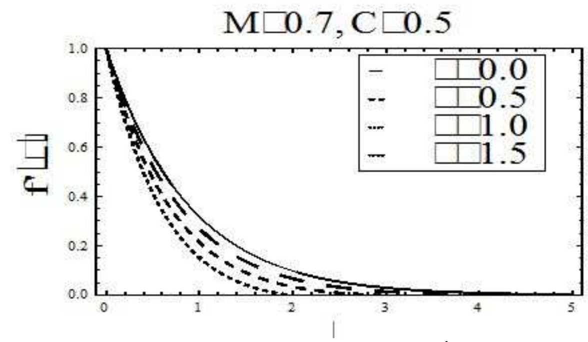

Figure. 2(a) Influence of λon $f'(\eta)$.

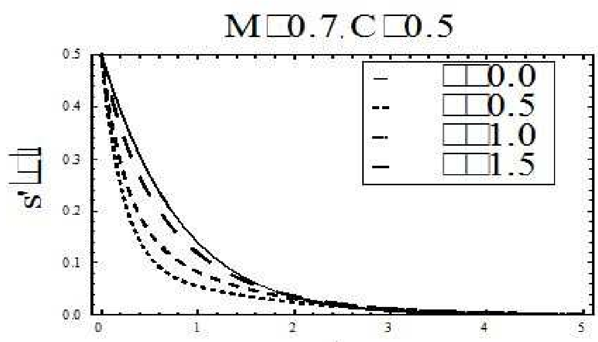

Figure. 2(b) Influence of λ on $s'(\eta)$

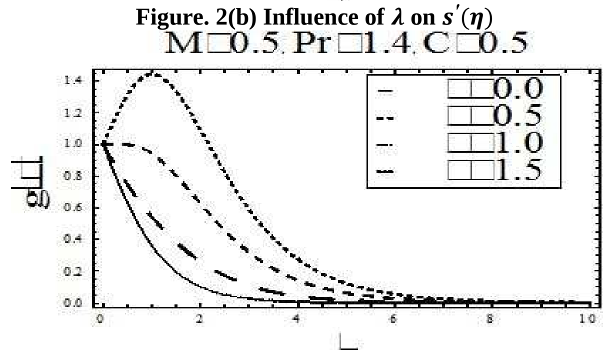

Figure. 2(c) Influence of λon $g(\eta)$.

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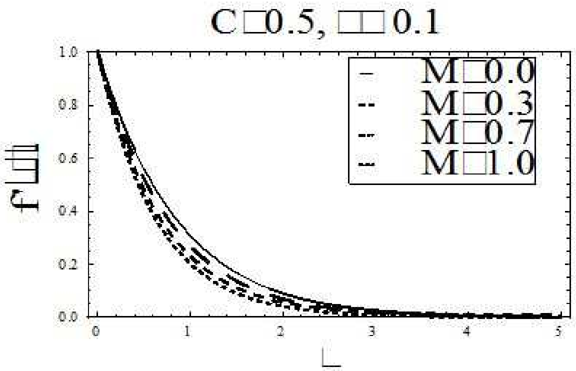

Figure. 3(a) Influence of Mon $f'(\eta)$.

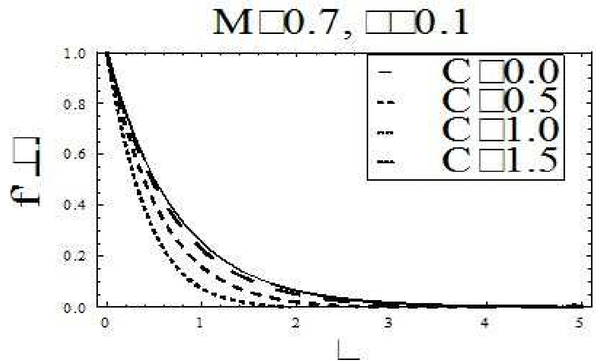

Figure. 4(a) Influence of C on $f'(\eta)$

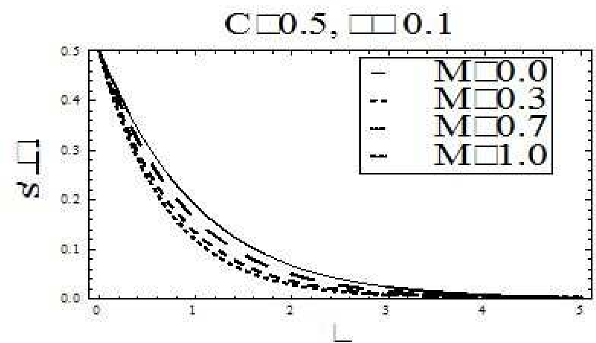

Figure. 3(b) Influence of M on $s'(\eta)$.

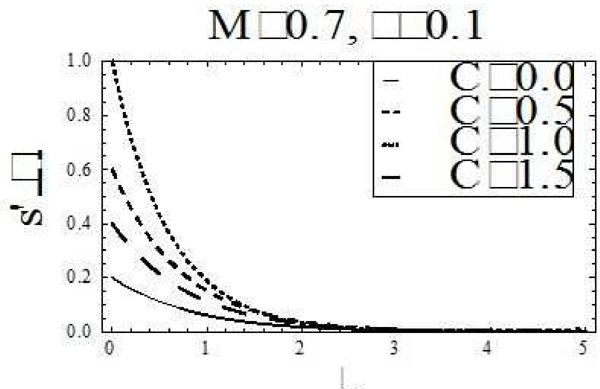

Figure. 4(b) Influence of Con $s'(\eta)$.

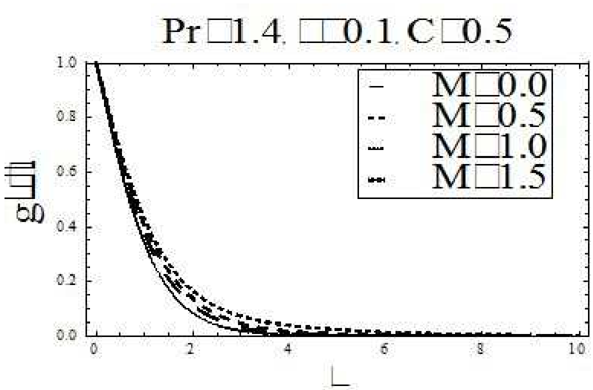

Figure. 3(c) Influence of λon $g(\eta)$.

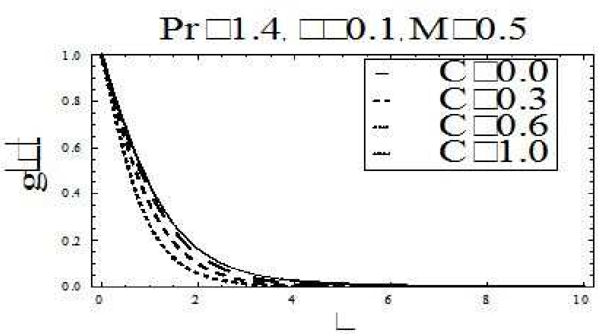

Figure. 4(c) Influence of C on $g(\eta)$

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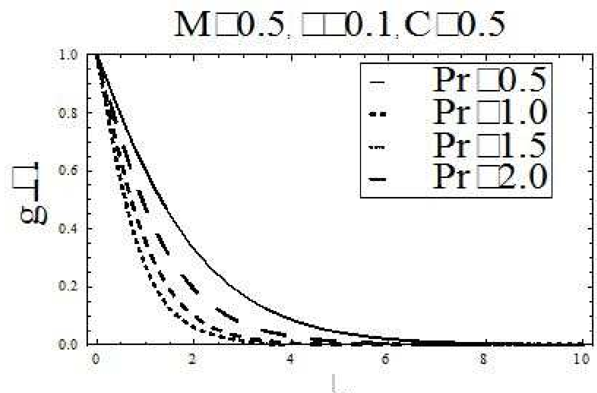

Figure. 5 Influence of Pr on $g(\eta)$

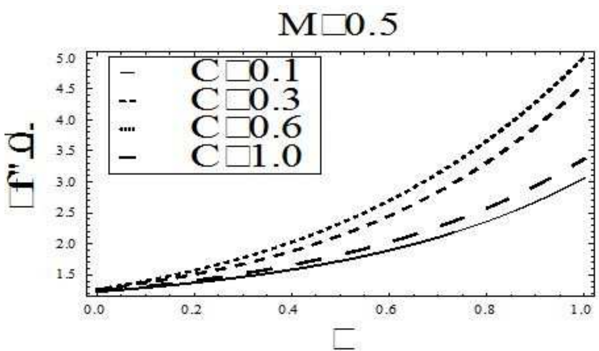

Figure. 6(a) Effect of Con $-g''(\eta)$.

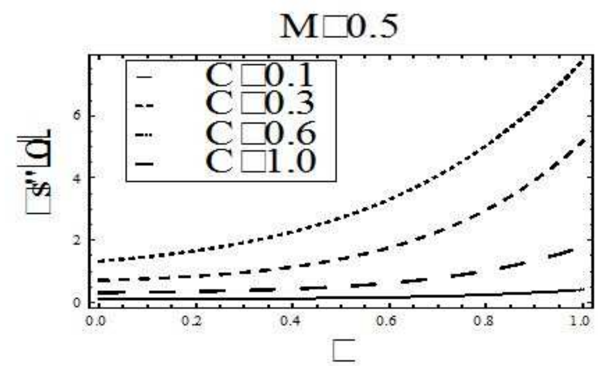

Figure. 6(b) Effect of Con $-s''(\eta)$.

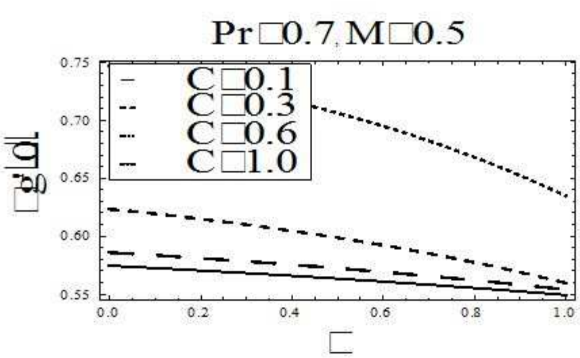

Figure.6(c) Effect of Con $-g'(0)$.

Figure 5 shows the effect of Prandtl number on the temperature profiles for a fixed values of M, λ and C. Here again increase in Prandtl number decreases the thermal boundary layer thickness. The effect of viscoelastic parameter 'λ' on the skin friction and heat transfer coefficients are plotted for various values of Stretching ratio 'c' in figures 6(a), 6(b) and 6(c), respectively.For a given value of 'c', the increase in λ increases the skin friction rates and the heat transfer coefficients.

Conclusions

In this study, the flow and heat transfer of a three dimensional viscoelastic fluid is studied in the presence of applied magnetic field. Using Homotopy Analysis Method, an approximate analytical solution is obtained in the form of Taylor series expansion in powers of the homotopy parameter 'h'. The effects of various parameters such as the magnetic parameter M; the stretching ratio 'c'; the viscoelastic parameter $\lambda$and the Prandtl numberPr on the velocity and temperature distributions as well as on skin friction and heat transfer coefficients are studied.It is observed that both velocity and boundary layer thicknesses decrease with increasing magnetic field. This shows that application of magnetic field causes a control on the boundary layer thickness. But the presence of magnetic parameter increases the heat transfer coefficients for both constant wall temperature and constant heat flux cases. The computations show that the skin friction and the heat transfer coefficient increase with the increasing viscoelastic parameter and the stretching parameter. The heat transfer coefficient forthe constant heat flux case is higher than that of the constant wall temperature case. Temperature and thermal boundary layer thickness are increasing for increase in the values of λ whereas the effect is opposite for boundary layer velocities.

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